Math 2130 - Homework # 2

Partial Derivatives

Part 1 - Partial derivatives

1. Find the first partial derivatives of the following functions.
   (a) $f(x, y) = xe^y$
   (b) $f(x, y) = 4x^3y^2 + 3x^2y^3 + 10$
   (c) $g(x, y) = e^{x^2y}$
   (d) $f(s, t) = \dfrac{s - t}{s + t}$
   (e) $h(x, y) = x\cos(xy)$
   (f) $f(x, y) = x^3ye^{2x^3}$
2. Find the four second partial derivatives of the following functions.
   (a) $f(x, y) = x^3 + xy^2 + 1$
   (b) $f(x, y) = x^3e^{2y}$
   (c) $f(x, y) = y\sin(xy)$
   (d) $g(s, t) = \ln(st)$
3. Let $h(x, y, z) = \cos(x + y + z^2)$. Find $\frac{\partial h}{\partial x}$, and $\frac{\partial^2 h}{\partial x \partial z}$.
4. Let $h(x, y) = \sqrt{x^2 + 3y}$. Find h_{xx} and h_{xy}.

Part 2 - Chain rule

5. Let $z = x\sin(y)$, $x = t^2$, and $y = 4t^3$.
   Find $\frac{dz}{dt}$.

6. Let $z = \sqrt{r^s + s^2}$, $r = \cos(2t)$, and $s = \sin(2t)$.
   Find $\frac{dz}{dt}$ evaluated at $t = \pi$.

7. Let $z = \sin(2x + y)$ where $x = s^2 - t^2$ and $y = s^2 + t^2$.
   Find z_s and z_t.

8. Let $z = e^{x+y}$ where $x = st$ and $y = s + t$.
   Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$. Evaluate both at $s = 1$ and $t = -1$.

9. Let $w = \sqrt{x^2 + 3y + z^3}$ where $x = st$, $y = rs$, and $z = rt$.
   Find $\frac{\partial w}{\partial r}$ at $s = 1$, $r = 1$, $t = 0$.
